647510 **BSc5MatC503x** **Seat No :________**

# B.Sc. Semester - 5 (Remedial) (*CBCS*) Examination
## March/April-2024 (NEW COURSE)
## Boolean Algebra & Complex Analysis -1(Core)

**Time: 2:30 Hours** **Marks: 70**

**Instructions:**

1. All questions are compulsory.
2. Figures to the right indicate marks.

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Que-1(A) Answer any two: (10)

(1) Define: Lattice as a partial ordered set. If $(L, \leq)$ is a lattice and $a, b, c \in L$ then prove that:

**(i)** $a * (b * c) = (a * b) * c$ and $a \oplus (b \oplus c) = (a \oplus b) \oplus c$

**(ii)** $a * (a \oplus b) = a$ and $a \oplus (a * b) = a$

(2) Define: Equivalence relation. Let $\mathbb{N}$ be the set of all natural numbers and R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a,b)R(c,d) \Leftrightarrow ad(b+c) = bc(a+d)$, check whether R is an equivalence relation on $\mathbb{N} \times \mathbb{N}$ or not.

(3) Prove that every chain is a distributive lattice.

Que-1(B) Answer any one: (04)

(1) Explain representation of Hasse diagram. Draw the Hasse diagram of partial order set $(P(A), \subseteq)$, where $A = \{a, b, c\}$.

(2) Define: (i) Bounded lattice (ii) Complemented lattice.
Give an example of a bounded lattice which is not a complemented lattice.

Que-2(A) Answer any two: (10)

(1) Define: Atoms in Boolean algebra.

(i) Prove: If $a$ and $b$ are distinct atoms of Boolean algebra $(B, *, \oplus, ', 0, 1)$ then $a * b = 0$.

(ii) Prove: If $x$ is a non-zero element of finite Boolean algebra $(B, *, \oplus, ', 0, 1)$ then there exists an atom $a \in B \ni a \leq x$.

(2) State and prove Stone's representation theorem for Boolean algebra.

(3) Define: Boolean expressions.
Express the Boolean expression $\alpha(x,y,z) = x \oplus y$ as the sum of product canonical form.

Que-2(B) Answer any one: (04)

(1) Define minterms in n-variables. Prove that the sum of all minterms of n variables $x_1, x_2, \ldots x_n$ is 1.

(2) Obtain the minimal sum of product of the Boolean expression
$\alpha(x,y,z) = xyz + xyz' + x'yz' + x'y'z$ by Karnaugh Map.

Que-3(A) Answer any two: (10)

(1) If $f(z) = u(x,y) + iv(x,y)$ is a complex function and $z_0 = x_0 + iy_0$ and $w_0 = u_0 + iv_0$ are fixed complex constants and $z = x + iy$ is complex variable then prove that

$$\lim_{z \to z_0} f(z) = w_0 \text{ iff } \lim_{(x,y) \to (x_0,y_0)} u(x,y) = u_0 \text{ and } \lim_{(x,y) \to (x_0,y_0)} v(x,y) = v_0.$$

(2) Obtain Cauchy-Riemann conditions for an analytic function $f(z)$ in polar form.

(3) Prove that $u = \left(r - \frac{1}{r}\right) sin\theta$ is harmonic. Find its conjugate harmonic.

Que-3(B) Answer any one: (04)

(1) If the complex function $f(z) = u + iv$ and its conjugate $\bar{f}(z) = u - iv$ are analytic functions then prove that $f(z)$ is constant function.

(2) If $u$ and $v$ are conjugate harmonic functions of $x$ and $y$ and $P = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$ and $Q = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$ then prove that $f(z) = P + iQ$ is analytic function.

Que-4(A) Answer any two: (10)

(1) In usual notations, prove that $\left|\int_a^b f(z)dz\right| \leq \int_a^b |f(z)|dz$.

(2) State and prove Cauchy's integral formula.

(3) Define: Contour. In usual notation prove that $\int_c f(z)dz = ir_0 \int_0^{2\pi} f(z_0 + re^{-i\theta})e^{i\theta} d\theta$, where $C: z - z_0 = r_0 e^{i\theta} and\ hence\ deduce\ that \int_c \frac{dz}{z-z_0} = 2\pi i.$

Que-4(B) Answer any one: (04)

(1) Find $\int_c \ f(z)dz$ where $f(z) = \begin{cases} 4y, & y > 0 \\ 1, & y < 0 \end{cases}$
where $C$ is a part of curve $x = y^3$ from points $z = -1 - i$ to $z = 1 + i$.

(2) Evaluate : $\int_c \ (z - 1)\, dz$ where $C$ is the arc from $z = 0$ to $z = 2$ consisting of the semi-circle $z = 1 + e^{i\theta}, \pi \leq \theta \leq 2\pi$.

Que-5(A) Answer any two: (10)

(1) State Cauchy-inequality and state and prove Liouville's theorem.

(2) State and prove Morera's theorem.

(3) State Cauchy's integral formula for derivatives and if $g(z_0) = \int_c \ \frac{z^2-2z+5}{z-z_0} dz, C: |z - 1| = 2$ then find $g(i), g(0), g(2), g'(2), g'(i), g(10 + i)$.

Que-5(B) Answer any one: (04)

(1) Evaluate$\int_c \ \frac{1}{(z^2+4)^2} dz, C: |z - i| = 2.$

(2) Find $\int_c \ \frac{z^3+2}{(z+i)^3} dz$, $C$: $|z|$=2

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